Tooth Tip Losses in External Gear Pumps

by

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NOMENCLATURE

Symbol	Definition
a	Addendum of gear tooth (in.)
$\bar{a}$	a/R
b	Constant
E	Efficiency loss at the tooth tip
$E(\%)$	Percentage efficiency loss
F	Force (lb_f)
F_o	Force acting on the tooth at $y = 0$ (lb_f)
h	Film thickness or clearance between the tip of a tooth and the casing (in.)
$\bar{h}$	h/R.
$\bar{h}_1$	Value of $\bar{h}$ giving minimum average loss
ℓ	Length of tooth tip (in.)
$\bar{\ell}$	ℓ/R
L	Length of choke in hydrostatic bearing (in.)
M_1	$50/(1+b/N)$
M_2	$(1+N/2)^2$
n	Number of teeth on one gear adjacent to the casing.
$\bar{n}$	n/N
N	Total number of teeth on one gear.
P	Delivery pressure of the pump (lb/in^2)
P_1	Recess pressure in hydrostatic bearing (lb/in^2)
P_o	Inlet pressure in hydrostatic bearing (lb/in^2)

Symbol	Definition
Q	Leakage flow rate for a hydrostatic bearing (in^3/s).
Q_1	Flow rate past the tip of the tooth (in^3/s)
Q_2	Flow rate due to the rotation of the gear (in^3/s),
Q_T	Total flow rate past the tooth tip (in^3/s).
r	Radius of choke of hydrostatic bearing (in.)
R	Pitch circle radius of the gear (in.)
R_O	Radius of the recess of hydrostatic bearing (in).
R_1	Outer radius of the hydrostatic bearing (in.)
S	Operating parameter
S_o	Sommerfeld number
S_1, S_2	Specific values for operating parameter S
$\bar{S}$	S_2/S_1
t	Thickness of fluid film in hydrostatic bearing (in.)
u	Velocity of fluid flow through the tip and casing clearance (in/s)
v	Tip velocity of the gear teeth (in/s)
V	Volume of fluid delivered by the gear pump per revolution (in^3)
w	Width of the gear (in.)
W	Axial thrust load (lb_f)
y	Distance from the tooth tip surface (in.)
β_1	Leakage power loss for hydrostatic bearing (in. lb_f/s)
β_2	Viscous power loss for hydrostatic bearing (in. lb_f/s)
μ	Viscosity of the fluid ($lb_f.s/in^2$)
T	Shear stress (lb_f/in^2)

T_o	Shear stress at y = 0 (lb_f/in^2)
ϕ	Total power loss of the gear pump (in. lb_f/s)
ϕ_1	Leakage power loss of the gear pump (in.lb_f/s)
ϕ_2	Viscous power loss of the gear pump (in.lb_f/s)
ϕ_P	Power delivered per tooth space of a gear pump (in. lb_f/s)
ω	Angular velocity of rotation (l/s)

TOOTH TIP LOSSES IN EXTERNAL GEAR PUMPS

ABSTRACT

Little information is available on the techniques applicable to the design of positive displacement machinery and to some extent this is probably due to the element of competition between individual manufacturers. The present work is concerned with a design procedure applicable to simple external gear pumps and attention is focussed on the power losses which occur in the region of the thin fluid film between the moving tips of the teeth of the gears and the adjacent stationary casing surface.

A method of obtaining the value of optimum clearance between these two members is derived and use of this value then ensures that the power loss in this region is a minimum. The results are expressed in terms of a non-dimensional operating parameter, based on the geometry of the machine and its operating conditions, the total number of teeth on each of its gears and a tooth tip clearance ratio.

Contours of constant power loss are also presented for pumps having different total numbers of teeth and a method of obtaining the minimum average power loss for a machine working over a range of the operating parameter is derived.

CHAPTER 1

INTRODUCTION

In the design of hydrostatic pumps and motors it is often possible to derive an optimum value of the mean, effective clearance between adjacent, moving surfaces, which separate regions of high and low pressure. Use of this optimum value of the clearance then ensures that the total of the viscous leakage power loss and the viscous drag power loss is a minimum which in turn leads to machine designs having high overall efficiencies. This approach has already been used in the design of axial piston pumps and motors in connection with the film thickness of their hydrostatic, slipper pad bearings (1)* and also for the clearances between their pistons and cylinders and the valve plate and rotor (2).

The present work concerns an external gear pump design problem associated with the derivation of the optimum value of the clearance between the tips of the teeth of the gears and the pump casing. Such pumps, in addition to their almost universal use in automobiles, are widely used in the aircraft and machine tool industries and also many other fields associated with fluid power engineering.

1.1 General Optimum Loss Conditions in Hydrostatic Equipment:

To illustrate the optimum loss conditions in hydrostatic equipment, the two power losses associated with the hydrostatic bearing shown in Fig. 1 (a list of figures is given in Appendix 2).

*(_) Underlined numbers in parenthesis refer to references given in appendix 1.

may be considered. The total power loss in such a bearing is made up of the leakage flow loss (i.e., the power required to pump the lubricant radially outward through the film space) and the viscous drag loss between the adjacent rotating and stationary surfaces.

The leakage flow rate, Q, from the central recess is readily shown (e.g. (3), (4)) to be given by

$$Q = \frac{P_1 \ \pi \ t^3}{6\mu \ \ell n \ (R_1/R_o)}$$

where μ is the viscosity of fluid.

This produces a leakage power loss β_1, equal to $P_o Q$ which may be written as:

$$\beta_1 = K_1 t^3$$

where K_1 is a constant for any given system configuration and set of operating conditions. At the same time another power loss, β_2, will exist due to the viscous drag between the two adjacent bearing surfaces one of which is stationary and the other rotating at an angular velocity of ω. The friction torque between these surfaces is readily shown (e.g. (3), (4)) to be

$$2\pi\mu\omega[R_1^4/4 - R_o^4/4]/t$$

and the power loss, β_2, due to this torque is given by

$$\beta_2 = 2\pi\mu\omega^2 [R_1^{4}/4 - R_o^{4}/4]/t$$

which may be written as:

$$\beta_2 = K_2/t$$

where K_2 is a constant for any given system. The total power loss will be $(\beta_1 + \beta_2) = K_1 t^3 + K_2/t$ and this can be minimized by a suitable selection of the bearing's film thickness, t.

It can be seen that if a large value of t is selected then the leakage power loss, $K_1 t^3$, will become excessive and similarly, if a very small value of t is selected then the viscous drag power loss, K_2/t, will become excessive. As a result there will be an optimum film thickness that will produce a minimum total of leakage and viscous power losses and this thickness can be evaluated by determining the minimum point of the curve representing the sum of the two power losses β_1 and β_2 as shown in Figure 2.

1.2 The External Gear Pump:

A similar condition exists for the optimum clearance between the tips of the teeth and the casing of the external gear pump and a simplified diagram of such a pump is shown in Figure 3a.

The pump consists of two gears closely fitted in a casing. The fluid is carried around the periphery of the revolving gears from the suction port to the discharge port. The meshing of the teeth between the two gears prevents the fluid from returning to the suction port from the discharge port.

The pressures range for such pumps is from a few hundred psi up to a thousand p.s.i. and in some cases even higher.

Gear pumps for hydraulic service have been built for capacities ranging from a fraction of a gallon per minute (g.p.m.) to 100 g.p.m. and higher. They may be directly driven by electric motor or internal combustion engines at speeds from 900 to 3600 rpm. For industrial applications speeds of 1200 to 1800 rpm. are very common for capacities up to 20 gpm. Further details are available in reference 3.

CHAPTER 2

POWER LOSSES ASSOCIATED WITH THE GEAR TEETH TIPS AND THE CASING:

There are two losses present at the tips of the pressurised teeth of the external gear pump such as that shown in Fig. 3a and it is assumed that both are associated with viscous type flow. One is due to a leakage flow rate back past the tip and the other is due to a viscous drag on the tooth tip surface of length ℓ.

In the analysis which follows it will be assumed that the viscosity of the working fluid is constant at some mean effective value determined by the operating temperature of the machine and its delivery pressure. This approximation has been used previously (1), (2) and has been found to give satisfactory results.

The losses may be evaluated as follows.

2.1 Leakage Flow Loss:

The flow rate, Q_1, past the tooth tip shown in Fig. 3b due to the pressure difference, P/n is given by:

$$Q_1 = \frac{wP\,h^3}{12n\ell\mu}$$

where h is the clearance between the tip of the teeth and the casing.

w is the width of the gear

P is the pressure rise across the pump

and n is the number of teeth on one gear adjacent to the casing. It should be noted that this quantity, n, may fluctuate during operation by an amount equal to unity but for simplicity it will be assumed constant. It has been assumed that the pressure drop across each tooth tip is equal to P/n and this has been suggested to be the case by Ernst (3).

The flow rate, Q_2, due to the rotation of the gear is given by:

$$Q_2 = wvh/2$$

where v is the velocity of the rotating teeth.

But $v = \omega (R + a)$

where R is the pitch circle radius of the gear and a is the addendum of the gear.

So that $v = \omega R (1 + \bar{a})$

where $\bar{a} = \frac{a}{R}$ and hence

$$Q_2 = -w\omega R(1+\bar{a})h/2$$

The total flow rate, Q_T, past the tooth tip is, therefore, given by:

$$Q_T = Q_1 + Q_2$$

or $$Q_T = \frac{wPh^3}{12n\ell\mu} - \frac{w\omega R(1+\bar{a})h}{2}$$

$$= w \left\{\frac{Ph^3}{12n\ell\mu} - \frac{\omega R(1+\bar{a})h}{2}\right\}$$

The leakage power loss, ϕ_1, associated with this flow rate is PQ_T/n where P is the pressure difference between the delivery and suction ports.

Hence $\phi_1 = \frac{wP}{n}\left[\frac{Ph^3}{12n\ell\mu} - \frac{\omega R(1+\bar{a})h}{2}\right]$

$$= \frac{wP^2}{n^2\mu}\left[\frac{h^3}{12\ell} - \frac{\omega\mu R(1+\bar{a})hn}{2P}\right]$$

or $$\phi_1 = \frac{wP^2R^3}{12\mu\bar{\ell}\,n^2}\left[\bar{h}^3 - \frac{6\mu\omega\bar{\ell}\bar{h}n(1+\bar{a})}{P}\right] \qquad (1)$$

where $\bar{\ell} = \ell/R$ and $\bar{h} = h/R$

2.2 Viscous Drag Loss:

The velocity, u, of the fluid flow through the tip and casing clearance is given by:

$$u = \frac{1}{2\mu}\frac{dP}{dx}(y^2 - hy) + \frac{v}{h}(h-y)$$

Differentiating u with respect to y gives

$$\frac{du}{dy} = \frac{1}{2\mu}\frac{dP}{dx}(2y-h) - \frac{v}{h}$$

Now by definition the shear stress T is given by

$$T = \mu\frac{du}{dy}$$

and substituting for du/dy gives

$$T = \mu\left[\frac{1}{2\mu}\frac{dP}{dx}(2y-h) - \frac{v}{h}\right]$$

or $$T = \frac{dP}{dx}\left(y - \frac{h}{2}\right) - \frac{\mu v}{h}$$

But the shearing stress, T_o, along the tip of the gear tooth (at y = 0) see Fig. 3b) is given by

$$T_o = -\frac{dP}{dx}\frac{h}{2} - \frac{\mu v}{h}$$

and since F = T x (area), where F is the force

$$\therefore \quad F_o = T_o \, \ell w$$

$$= -\frac{P}{\ell n}\frac{h}{2}\ell w - \mu\frac{v}{h}\ell w$$

or $F_o = -w\left\{\frac{Ph}{2n} + \frac{\mu v \ell}{h}\right\}$

Now, as before, $v = \omega(R + a)$ and the viscous drag power loss, ϕ_2, due to F_o is, therefore, given by:

$$\phi_2 = \omega\,(R + a)\,F_o$$

$$= \omega\,(R + a)\,w\left\{\frac{Ph}{2n} + \frac{\mu\ell\omega(R + a)}{h}\right\}$$

$$= \omega\,wR(1 + \bar{a})\left\{\frac{Ph}{2n} + \frac{\mu\ell\omega R(1+\bar{a})}{h}\right\}$$

or $$\phi_2 = \omega w R^2(1+\bar{a})\left\{\frac{P\bar{h}}{2n} + \frac{\mu\bar{\ell}\omega(1+\bar{a})}{\bar{h}}\right\} \qquad (2)$$

2.3 The Total Power Loss and Delivery Power of the Pump

The total power loss ϕ is the sum of the leakage loss ϕ_1 and the viscous drag loss ϕ_2 so that

$$\phi = \phi_1 + \phi_2$$

$$= \omega w R^2\,(1 + \bar{a})\left\{\frac{P\bar{h}}{2n} + \frac{\mu\omega\bar{\ell}(1 + \bar{a})}{\bar{h}}\right\}$$

$$+ \frac{wP^2R^3}{12\mu\ell n^2}\left\{\bar{h}^3 - \frac{6\mu\omega\bar{\ell}\bar{h}n(1+\bar{a})}{P}\right\}$$

$$= w\ \{\frac{\omega PR^2(1+\bar{a})}{2n}\ [\bar{h} + \frac{\mu\omega}{P}\ \frac{2n\bar{\ell}(1+\bar{a})}{\bar{h}}]$$

$$+ \frac{P^2R^3}{12\mu\ell n^2}\ [\bar{h}^3 - \frac{6\mu\omega}{P}\ \bar{\ell}\bar{h}n\ (1+\bar{a})]\}$$

Putting $\mu\omega/P = S_O$, which is a basic form of the Sommerfeld number of the system, the total power loss may be written as

$$\phi = w\ \{\frac{\omega PR^2(1+\bar{a})}{2n}\ [\bar{h} + \frac{2nS_O\bar{\ell}(1+\bar{a})}{\bar{h}}]$$

$$+ \frac{P^2R^3}{12\mu\ell n^2}\ [\bar{h}^3 - 6nS_O\bar{\ell}\bar{h}(1+\bar{a})]\}$$

This may be written as follows:

$$\phi = w\ \{\frac{\omega PR^2(1+\bar{a})}{2n}\ [\bar{h} + \frac{2nS_O\bar{\ell}(1+\bar{a})}{\bar{h}}]$$

$$+ \frac{P^2R^2S_O\bar{\ell}n(1+\bar{a})}{2\mu n^2\bar{\ell}}\ [\frac{\bar{h}^3}{6nS_O\bar{\ell}\ (1+\bar{a})} - \bar{h}]\}$$

$$= \frac{w\omega PR^2}{2n}\ \{(1+\bar{a})\ [\bar{h}+2nS_O\bar{\ell}(1+\bar{a})/\bar{h}]$$

$$+ \frac{PS_O(1+\bar{a})}{\mu\omega}\ [\frac{\bar{h}^3}{6nS_O\bar{\ell}(1+\bar{a})} - \bar{h}]\}$$

$$\text{or}\quad \phi = \frac{w\omega PR^2}{2n}\ \{\bar{h}(1+\bar{a}) + \frac{2nS_O\bar{\ell}(1+\bar{a})^2}{\bar{h}} + \frac{(1+\bar{a})\bar{h}^3}{6nS_O\bar{\ell}\ (1+\bar{a})} - \bar{h}(1+\bar{a})\}$$

Introducing a non-dimensional operating parameter S given by

$$S = 2nS_o\bar{\ell}\bar{a}$$

then gives

$$\phi = \frac{w\omega PR^2\bar{a}}{2n} \left\{\frac{S}{\bar{h}}\left(\frac{1+\bar{a}}{\bar{a}}\right)^2 + \frac{\bar{h}^3}{3S}\right\}$$

or $$\phi = \frac{w\omega PRa}{2n} \left\{ \frac{S}{\bar{h}}\left(\frac{1+\bar{a}}{\bar{a}}\right)^2 + \frac{\bar{h}^3}{3S}\right\}$$

However, this power loss at each tooth tip only occurs during the time which the tooth is adjacent to the casing and the ratio of this time to the total time of operation will be approximately $n/N = \bar{n}$ where N is the total number of teeth on one gear.

Hence the total power loss, ϕ_T, per tooth tip is given by:

$$\phi_T = \phi.\bar{n}$$

$$= \frac{w\omega\ PRa}{2N} \left[\frac{\bar{h}^3}{3S} + \frac{S(1+1/\bar{a})^2}{\bar{h}}\right] \qquad (3)$$

Hadekel (5) (page 70) has shown that the volume V, of fluid delivered by the pump per revolution is given by

$$V = 4\pi R\ aw(1 + b/N)$$

where $b \simeq 0.3$ for numbers of teeth between 6 and 8
and $b \simeq 0.27$ for numbers of teeth greater than 8.

Hence the power delivered, ϕ_P, by the pump per tooth space at a speed of ω is:

$$\phi_P = \frac{V}{2N} \cdot \frac{\omega}{2\pi} \cdot P$$

which may be written as

$$\frac{4\pi R\ a\ w\ (1 + b/N)\ \omega P}{2N.2\pi}$$

or

$$\phi_P = \frac{R\ a\ w\ (1 + b/N)\ \omega P}{N} \qquad (4)$$

2.4 The Efficiency Loss at a Tooth Tip:

The efficiency loss, E, at each tooth tip will be given by the ratio of ϕ_T to ϕ_P so that

$E = \phi_T/\phi_P$ = Power loss at tooth tip/Power delivered by inter-tooth space.

Using the values given by equations (3) and (4)

$$\therefore \phi_T/\phi_P = \frac{w\omega PRa}{2N} \left\{\frac{\bar{h}^3}{3S} + \frac{S(1+1/\bar{a})^2}{\bar{h}}\right\} \Big/ \frac{R\ a\ w\ (1 + b/N)\ \omega P}{N}$$

$$\text{or} \quad E = \left[\frac{\bar{h}^3}{3S} + \frac{S(1 + 1/\bar{a})^2}{\bar{h}}\right]/2(1+b/N)$$

The percentage efficiency loss, E(%), is then given by:

$$E(\%) = 100\ [\ \frac{\bar{h}^3}{3S} + \frac{S(1 + 1/\bar{a})^2}{\bar{h}}]/2(1+b/N)$$

or

$$E(\%) = 50\ [\frac{\bar{h}^3}{3S} + \frac{S(1 + 1/\bar{a})^2}{\bar{h}}]/(1+b/N)$$

For gears it is general practice to have $a = 2R/N$ and this may be written as $1/\bar{a} = N/2$, therefore

$$E(\%) = 50\ [\frac{\bar{h}^3}{3S} + \frac{S(1 + N/2)^2}{\bar{h}}]/(1+b/N) \qquad (5)$$

It should be noted that this equation may be written in the form of two terms, $K_3\bar{h}^3 + K_4/\bar{h}$, where K_3 and K_4 are constants for a given pump and is, therefore, similar to the power loss equation of the hydrostatic bearing discussed in Section 1.1. As a result if $\bar{h}$ is very large then the efficiency loss will become excessive and if it is made very small the efficiency will also become excessive. It follows that there will be some value of $\bar{h}$ at which the loss will be a minimum.

2.5 Conditions of Minimum Power Loss:

To obtain the condition of minimum power loss, equation (5) may be differentiated with respect to $\bar{h}$ and equated to zero, giving

$$\frac{dE(\%)}{d\bar{h}} = \frac{50}{(1+b/N)}\ [\frac{\bar{h}^2}{S} - \frac{S(1 + N/2)^2}{\bar{h}^2}] = 0$$

Hence, $\bar{h}^2/S - S(1+N/2)^2/\bar{h}^2 = 0$

or $\bar{h}^4 - S^2 (1 + N/2)^2 = 0$

giving $\bar{h} = \sqrt{S(1+N/2)}$ (6)

which is the value of $\bar{h}$ for the condition of minimum power loss.

Its variation with S is shown in Figure 4 for different total numbers of teeth, N, and from Equation (6) it is seen that the value of $\bar{h}$ varies with the parameter S and number of teeth N.

2.6 Contours of Constant Loss:

To obtain contours of constant power loss, it is convenient to transform equation (5) into a simple quadratic equation in S^2. This may be done by multiplying it by $S\bar{h}$ giving

$$E(\%)\ S\bar{h} = 50\ [\bar{h}^4/3 + S^2\ (1 + N/2)^2]/(1+b/N)$$

Writing $M_1 = 50/(1 + b/N)$ and $M_2 = (1 + N/2)^2$ gives

$$E(\%)\ S\bar{h} = M_1\ (\bar{h}^4/3 + M_2S^2)$$

and this may be written in the form

$$M_1M_2S^2 - E(\%)\ S\bar{h} + M_1\bar{h}^4/3 = 0$$

After dividing by M_1M_2 the equation becomes

$$S^2 - E(\%)\bar{h}S/M_1M_2 + \bar{h}^4/3M_2 = 0 \qquad (7)$$

Equation (7) is a quadratic equation in S where M_1 and M_2 are simple functions of the total number of teeth, N, and may be solved for given values E(%) and $\bar{h}$ and for different numbers of teeth.

The resulting variation of $\bar{h}$ with S for a given value of E(%) (with N constant) becomes a contour of constant loss and results showing these contours are given in Figures 5 to 8 inclusive. These contours of constant loss are almost straight lines for small values of S and $\bar{h}$ as is illustrated in these figures.

It follows from equation (6) that pumps may be designed for minimum tooth tip loss for any given configuration and for any value of S. In practice, however, it would generally be uneconomical for a manufacturer to design a pump for a specific value of S.

It follows that designs would be produced to cover a specific ranges of S and care would be taken to ensure that the losses would not be too great by reference to the contours.

As a result the design of a pump on a minimum power loss basis, would involve ensuring that the average loss for a range of S were a minimum and this condition is examined in the following section.

2.7 Minimum Loss Conditions with Variable Pump Parameter:

The results derived in the previous section apply only to machines working under constant operating conditions.

In practice however, it is often necessary to design a machine for a range of one or more of the values of speed, delivery pressure and fluid viscosity and as a result there will be a working range of the operating parameter S. If there is no indication of the value at which it will operate for the maximum period of its use then it is desirable that the average power loss over the entire operating range is a minimum and this can be achieved as follows.

Integrating equation (5) over a range of S from S_1 to S_2 gives

$$\int_{S_1}^{S_2} E(\%) = \frac{50}{(1+b/N)} \int_{S_1}^{S_2} \left[\frac{\bar{h}^3}{3S} + \frac{S(1 + N/2)^2}{\bar{h}} \right] dS$$

and after integration and inserting the limits this gives,

$$\int_{S_1}^{S_2} E(\%) = \frac{50}{(1+b/N)} \left[\frac{\bar{h}^3 \ln(S_2/S_1)}{3} + \frac{(1+N/2)^2 (S_2^2 - S_1^2)}{2\bar{h}} \right]$$

Introducing an operating parameter range ratio $\bar{S} = S_2/S_1$ then gives

$$\int_{S_1}^{S_2} E(\%) = \frac{50}{(1+b/N)} \left[\frac{\bar{h}^3 \ln\bar{S}}{3} + \frac{(1+N/2)^2 \; S_1^2 \; (\bar{S}^2 - 1)}{2\bar{h}} \right]$$

Now the average value of E(%) will be given by

$$\frac{\int_{S_1}^{S_2} E(\%)}{(S_2 - S_1)} = \frac{\int_{S_1}^{S_2} E(\%)}{S_1(\bar{S} - 1)}$$

and this is then given by

$$\text{Average } E(\%) = \frac{50}{(1+b/N)}\left[\frac{\bar{h}^3 \ell n\bar{S}}{3S_1(\bar{S}-1)} + \frac{(1+N/2)^2 S_1^2(\bar{S}^2-1)}{2\bar{h}S_1(\bar{S}-1)}\right]$$

$$= \frac{50}{(1+b/N)}\left[\frac{\bar{h}^3 \ell n\bar{S}}{3S_1(\bar{S}-1)} + \frac{(1+N/2)^2 S_1(\bar{S}+1)}{2\bar{h}}\right] \quad (8)$$

To determine the value of $\bar{h}$ which gives a minimum average value of E(%) equation (8) must be differentiated with respect to $\bar{h}$ and equated it to zero. This gives

$$\frac{d(\text{average } E(\%))}{d\bar{h}} = \frac{50}{(1+b/N)}\left[\frac{\ell n\bar{S}\ \bar{h}^2}{S_1(\bar{S}-1)} - \frac{(1+N/2)^2\ S_1(\bar{S}+1)}{2\bar{h}^2}\right] = 0$$

or $$\frac{\ell n\bar{S}\ \bar{h}^2}{S_1(\bar{S}-1)} - \frac{(1+N/2)^2\ S_1(\bar{S}+1)}{2\bar{h}^2} = 0$$

Multiplying by $\bar{h}^2$ gives

$$\frac{\ell n\ \bar{S}\ \bar{h}^4}{S_1(\bar{S}-1)} = (1+N/2)^2 S_1(\bar{S}+1)/2$$

and dividing by $\ell n\bar{S}/S_1(\bar{S}-1)$ gives

$$\bar{h}^4 = (1+N/2)^2\ S_1^2\ (\bar{S}+1)(\bar{S}-1)/2\ell n\bar{S}$$

This may be written as

$$\bar{h}_1 = \sqrt{S_1(1+N/2)}\ \sqrt{(\bar{S}^2 - 1)/2\ell n\bar{S}} \quad (9)$$

Where $\bar{h}_1$ is the value of $\bar{h}$ giving a minimum average loss when S varies from S_1 to S_2.

Equation (9) is plotted for given values of S_1, N and $\bar{S}$ in Figures 9 to 12 inclusive. From these curves it is possible to select the value of $\bar{h}$ giving the minimum average power loss for given values of S_1 and $\bar{S}$.

It is of interest to note that if the range of S is reduced to zero then at $\bar{S} = 1.0$, the value of $\bar{h}_1$ should correspond to that of $\bar{h}$ given by Equation (6). This may be confirmed by writing the ratio $\bar{S} = (1 + \varepsilon)$ where ε is a very small positive quantity.

Equation (9) is

$$\bar{h}_1 = \sqrt{S_1(1+N/2) \quad \sqrt{(\bar{S}^2 - 1)/2 \ \ln\bar{S}}}$$

and if $\bar{S} = (1 + \varepsilon)$, then $\ln\bar{S} = \ln(1 + \varepsilon) = \varepsilon$, as $\varepsilon \to 0$.
Equation (9) then becomes:

$$\bar{h}_1 = \sqrt{S_1(1+N/2) \quad \sqrt{1+2\varepsilon + \varepsilon^2 - 1/2\varepsilon}}$$

and if powers of ε greater than unity are neglected

$$\bar{h}_1 = \sqrt{S_1(1 + N/_2) \quad \sqrt{2\varepsilon/2\varepsilon}}$$

$$\text{or} \quad \bar{h}_1 = \sqrt{S_1(1 + N/2)}$$

which is the value of $\bar{h}$ given by Equation (6).

CHAPTER 3

DISCUSSION OF RESULTS AND CONCLUSIONS:

3.1 General

The results obtained in Chapter 2, Section 2.5, concerning the operation of an external gear pump at a constant value of the operating parameter, S, show that there is a unique value of the film thickness ratio, $\tilde{h}$, at which the total power loss in the region of the tips of the teeth of gears is a minimum (see figure 4). It can be seen from equation (6) that this value of $\tilde{h}$ increases with the square root of operating parameter and also increases with the total number of teeth, N, on each gear. This variation of $\tilde{h}$ with the total number of teeth is also shown in Fig. 4.

Failure to operate at or near the optimum value of the tooth tip clearance ratio can result in excessive values of efficiency loss and this is illustrated by the contours of constant loss in figures 5 to 8 inclusive.

It can be seen that losses of 10 to 20% or even greater may occur if an incorrect value of the tooth tip clearance were selected and this would result in an unacceptable design of a machine.

3.2 The Effect of the Total Number of Teeth

Increasing the total number of teeth on each gear increases the efficiency loss for a given value of the operating parameter. This is illustrated for example by

Figures 5 (N=8) and 7 (N = 16) where for a value of the operating parameter of say 4×10^{-7} the efficiency loss will be just under 0.5% for N = 8 but the efficiency loss will be greater than 1% for N = 16.

In general it may be said that gears having small (8 to 12) numbers of teeth are preferable to those having larger numbers of teeth (16 to 20). However, the ripple of the delivery from the pump will decrease as the number of teeth increases and it remains for the designer to select the number of teeth which gives him the best compromise between these two conflicting conditions.

3.3 The Effect of Variable Operating Conditions

As mentioned at the end of section 2.6 of Chapter 2 it is unlikely that machines would be manufactured for one specific value of the operating parameter. In general they would be designed and manufactured to cover a range of values of the operating parameter, S, say from S_1 to S_2. The condition for minimum average loss under this situation has been determined in Chapter 2, Section 2.7, and is perhaps best illustrated by considering the following example. Assume that the machine with 16 teeth per gear is to operate between two values of S, say $S_1 = 2 \times 10^{-7}$ and $S_2 = 20 \times 10^{-7}$. Then $\bar{S} = S_2/S_1 = 10$ and from equation (9) the value of tooth tip clearance ratio may be obtained. This value may then be substituted into equation (5) and the variation of the efficiency loss, as S increases from S_1 to S_2 may be determined and gives curve A of Figure 13. For comparison purposes this variation of the power or efficiency loss may be considered together with the power or efficiency loss variation of a different machine which has been designed to give minimum loss at S_1 (curve B Figure 13) and also with that of yet another machine designed to give minimum loss at S_2 (curve C Fig. 13). It is readily seen that although curves B and C give lower losses at their specified values of S (i.e., S_1 and S_2 respectively), their average value of loss over the entire range S_1 to S_2 is much greater than that of curve A. As a result, the optimisation procedure giving Curve A is of considerable value.

3.4 Conclusions

A method of evaluating the total power losses occurring at the tips of the teeth of a simple external gear pump has been derived and enables the conditions required for minimum power loss to be determined. It has been found that the total power loss is made up of two components, one of which is a leakage flow rate back past the tips of the teeth and the other of which is produced by the shearing of the thin fluid film between the tip of each gear tooth and the casing of the pump. It has also been found that the total of these two losses for a given pump configuration and a given set of operating conditions has a minimum at a specific value of the tooth tip clearance and an equation giving this value has been derived.

General results have been obtained in graphical form for machines having a total of 8, 12, 16 and 20 teeth per gear and the work has been extended to include the presentation of loss contours. These contours enable the efficiency loss due to tooth tip losses to be determined when a machine is not operating at its best efficiency point.

In addition an analytical study of the average power loss at the tips of the gear teeth when a machine is to operate over range of speed, delivery pressure, viscosity or even a combination of ranges of these quantities has enabled a minimum average power loss condition to be determined. Use of the results of this study applied to an example concerning the average loss for a machine working over a range of its operating parameter of 10 to 1 have shown that in a specific case its maximum loss need never exceed 3% whereas choice of an incorrect value of the tooth tip clearance ratio based on a constant value of the operating parameter can readily lead to losses equal to or greater than 6%.

It is considered that the results obtained in the present work will be of value to those concerned with the design of external gear pumps.

APPENDIX 1

REFERENCES

(1) Shute, N.A. and Turnbull, D.E., "Minimum Power Loss of Hydrostatic Slipper Bearings for Axial Piston Machines". Proc. I. Mech. E. Conf. Lub and Wear 1963, Paper 1.

(2) Shute, N.A. and Turnbull, D.E., "Minimum Power Loss Conditions of the Pistons and Valve Plate in Axial-Type Pumps and Motors", A.S.M.E. Paper No. 63-WA-90, 1963.

(3) Ernst, W., "Oil Hydraulic Power and its Industrial Applications", McGraw-Hill Book Co., New York, 2nd Ed. 1960.

(4) Fuller, Dudley, D., "Theory and Practice of Lubrication for Engineers", John Wiley & Sons, Inc. New York, 4th Ed., 1966.

(5) Hadekel, R., "Displacement Pumps and Motors", Sir Isaac Pitman & Sons, London, 1st Ed. 1951.

APPENDIX 2

LIST OF FIGURES

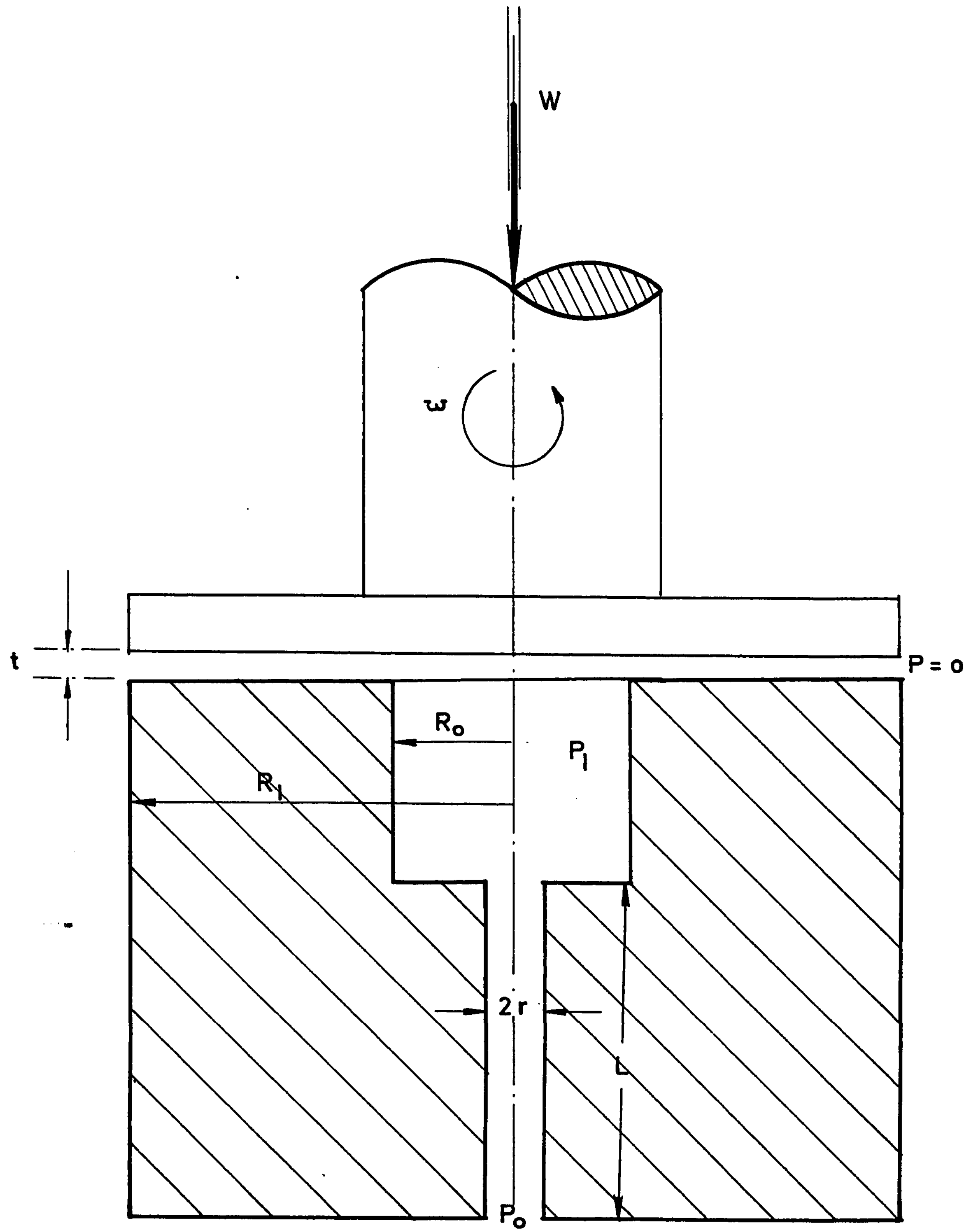

FIGURE 1 : HYDROSTATIC BEARING

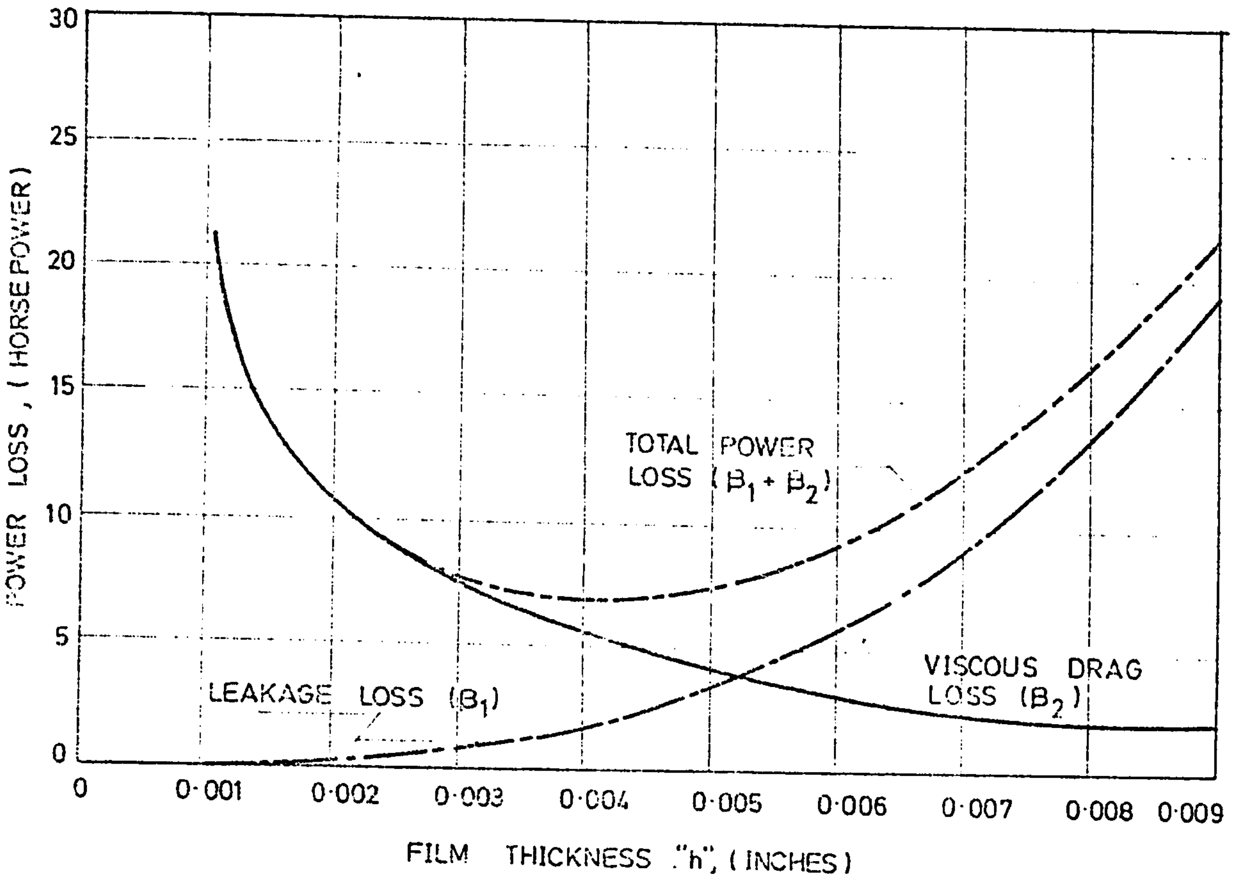

FIGURE 2_ CURVES SHOWING EFFECT OF FILM THICKNESS ON POWER LOSS IN HYDROSTATIC BEARING.

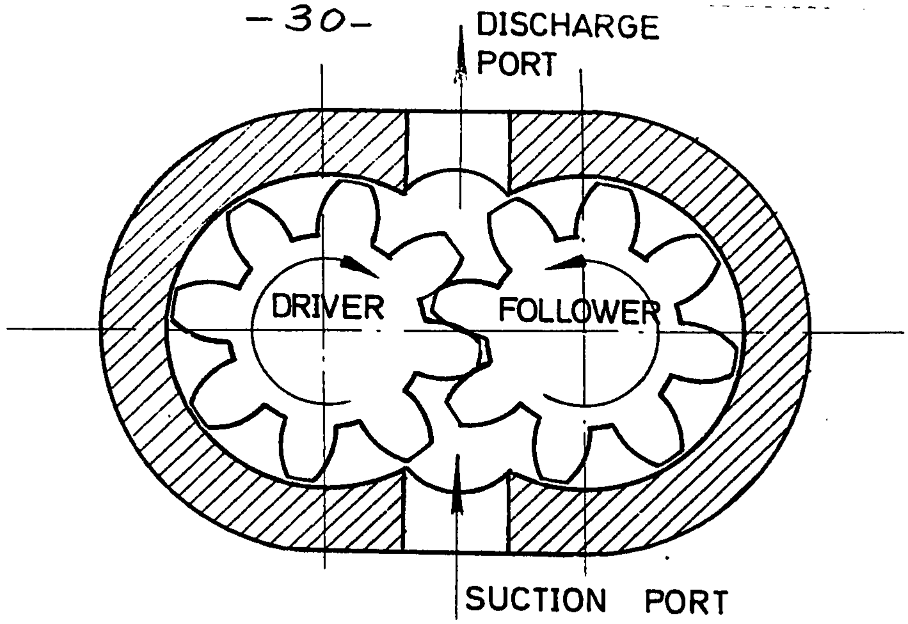

FIGURE 3a: SIMPLE EXTERNAL GEAR PUMP

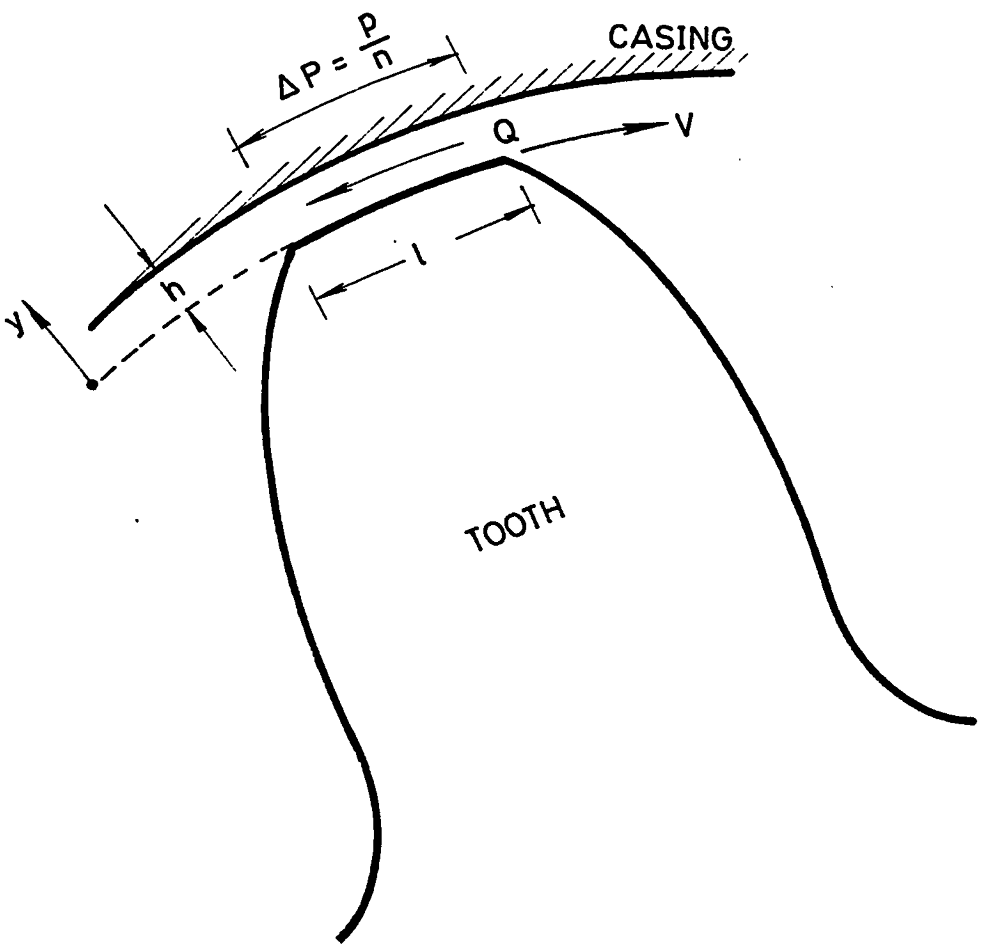

FIGURE 3b: CONFIGURATION OF TOOTH AND CLEARANCE

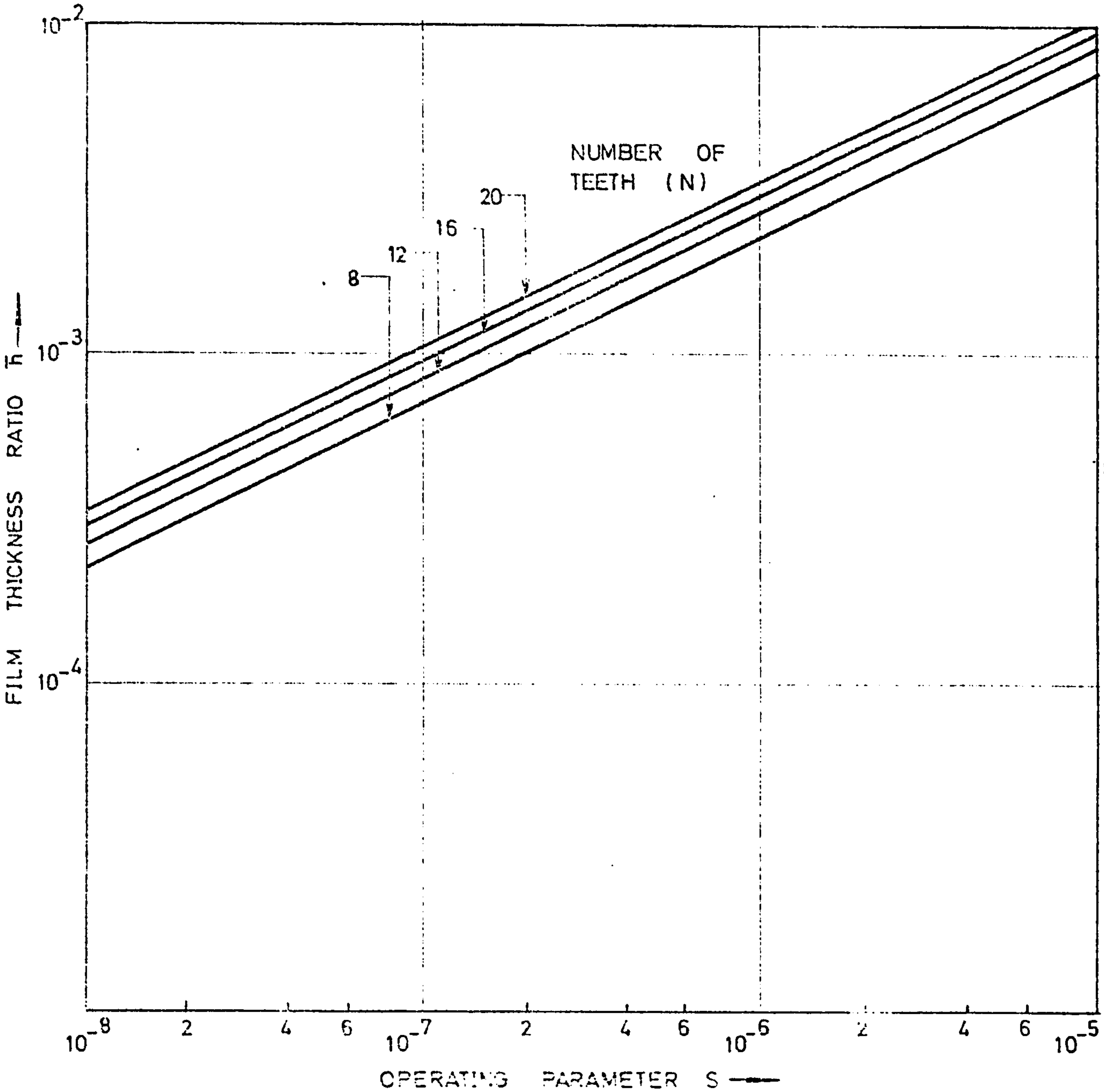

FIGURE 4- VARIATION OF FILM THICKNESS RATIO $\bar{h}$ WITH S FOR MINIMUM LOSS CONDITIONS.

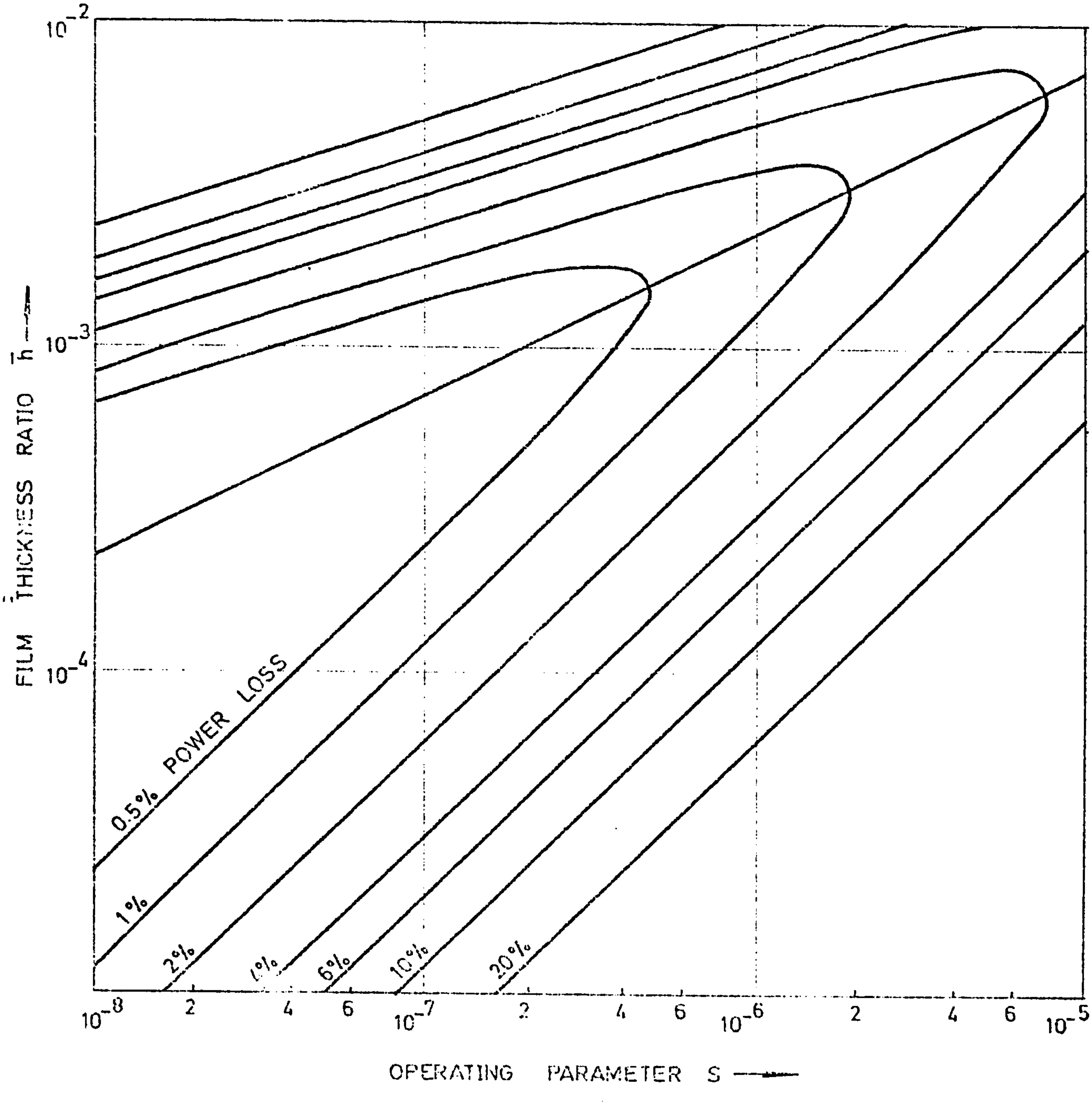

FIGURE 5 - CONTOURS OF LOSSES FOR N = 8

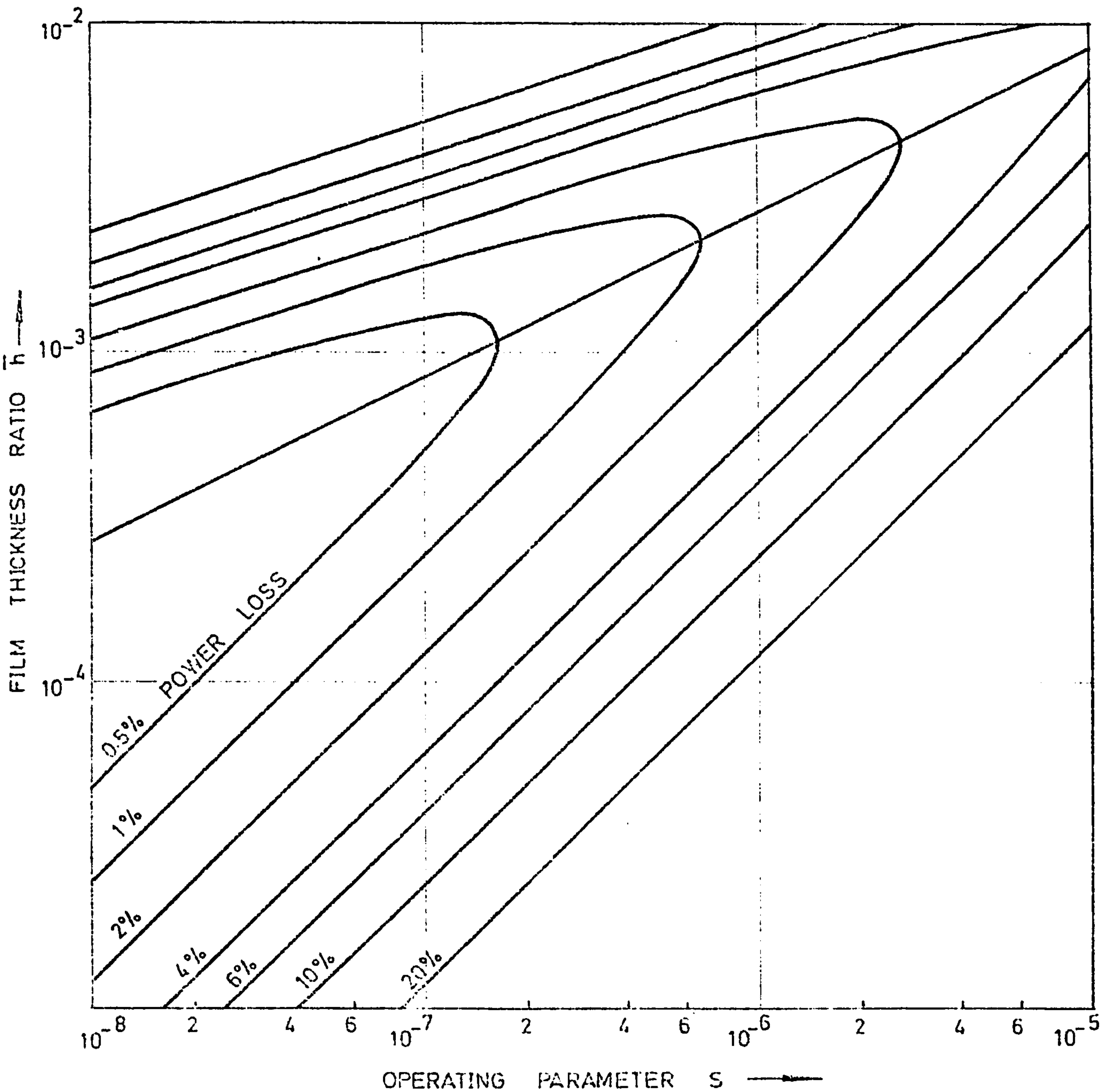

FIGURE 6– CONTOURS OF LOSSES FOR N = 12

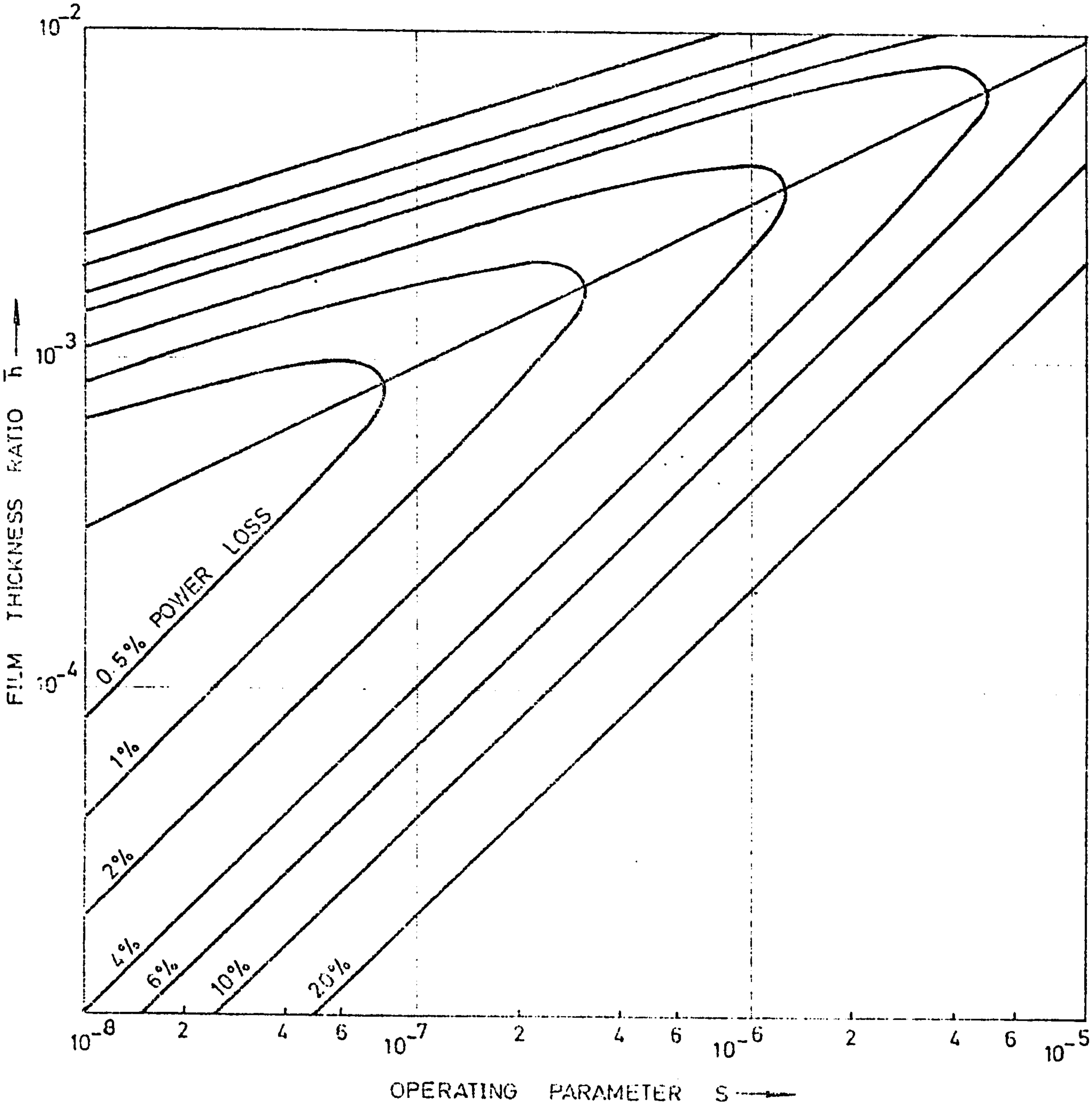

FIGURE 7_ CONTOURS OF LOSSES FOR N = 16

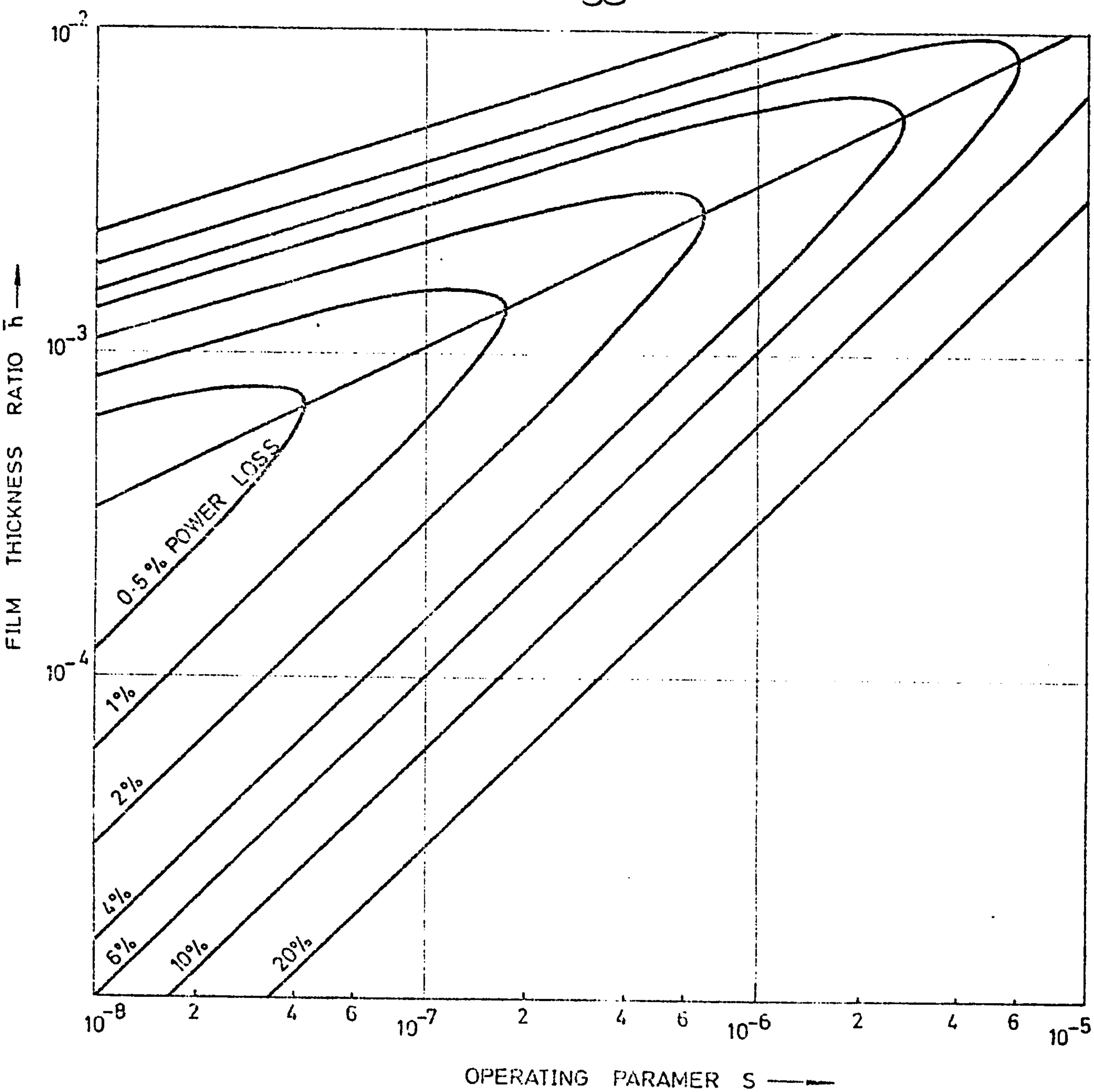

FIGURE 8. CONTOURS OF LOSSES FOR N = 20

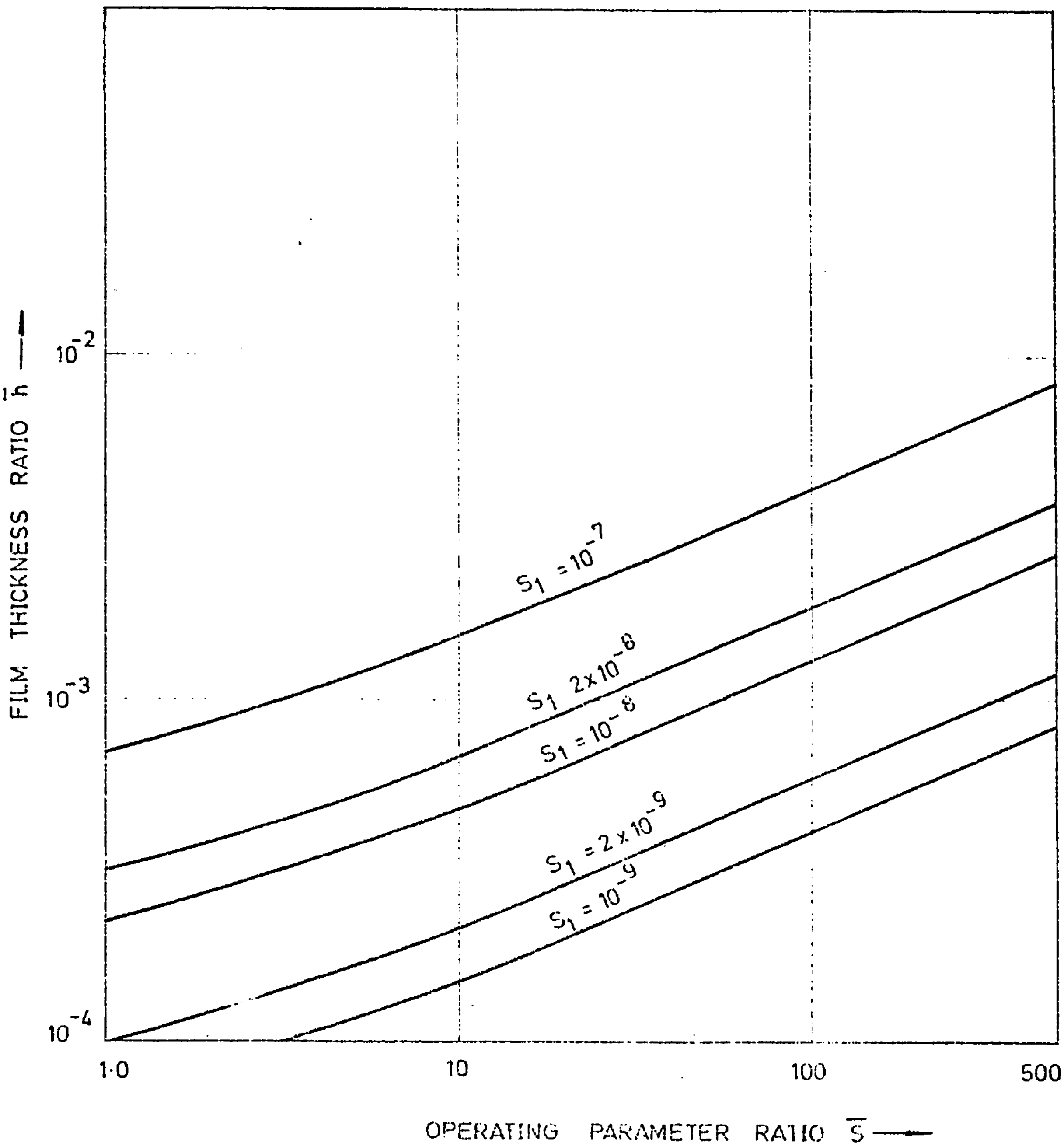

FIGURE 9 - VARIATION OF FILM THICKNESS RATIO $\bar{h}$ WITH OPERATING PARAMETER RATIO $\bar{S}$. (N = 8)

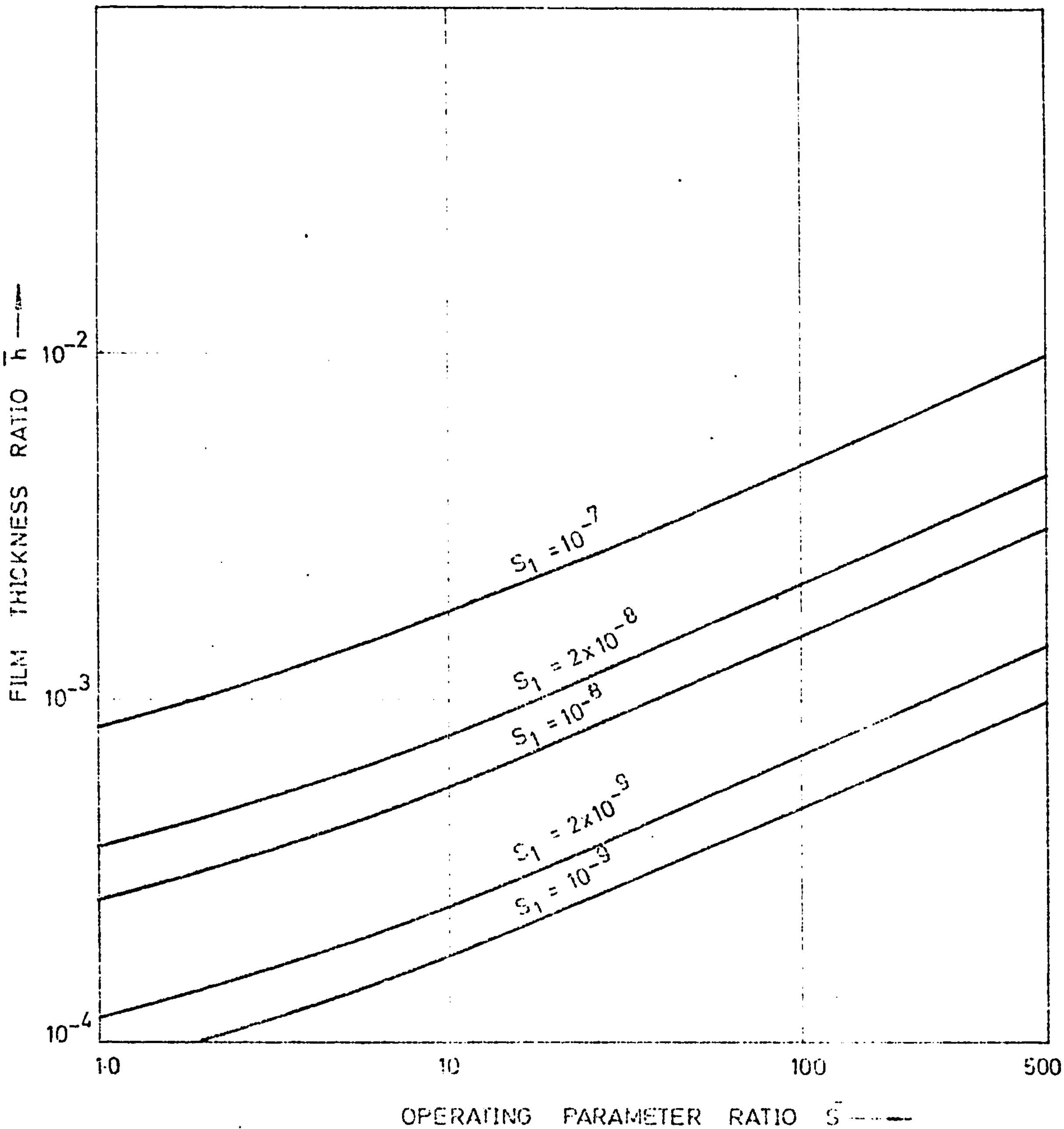

FIGURE 10 - VARIATION OF FILM THICKNESS RATIO $\bar{h}$ WITH OPERATING PARAMETER RATIO $\bar{S}$. (N = 12)

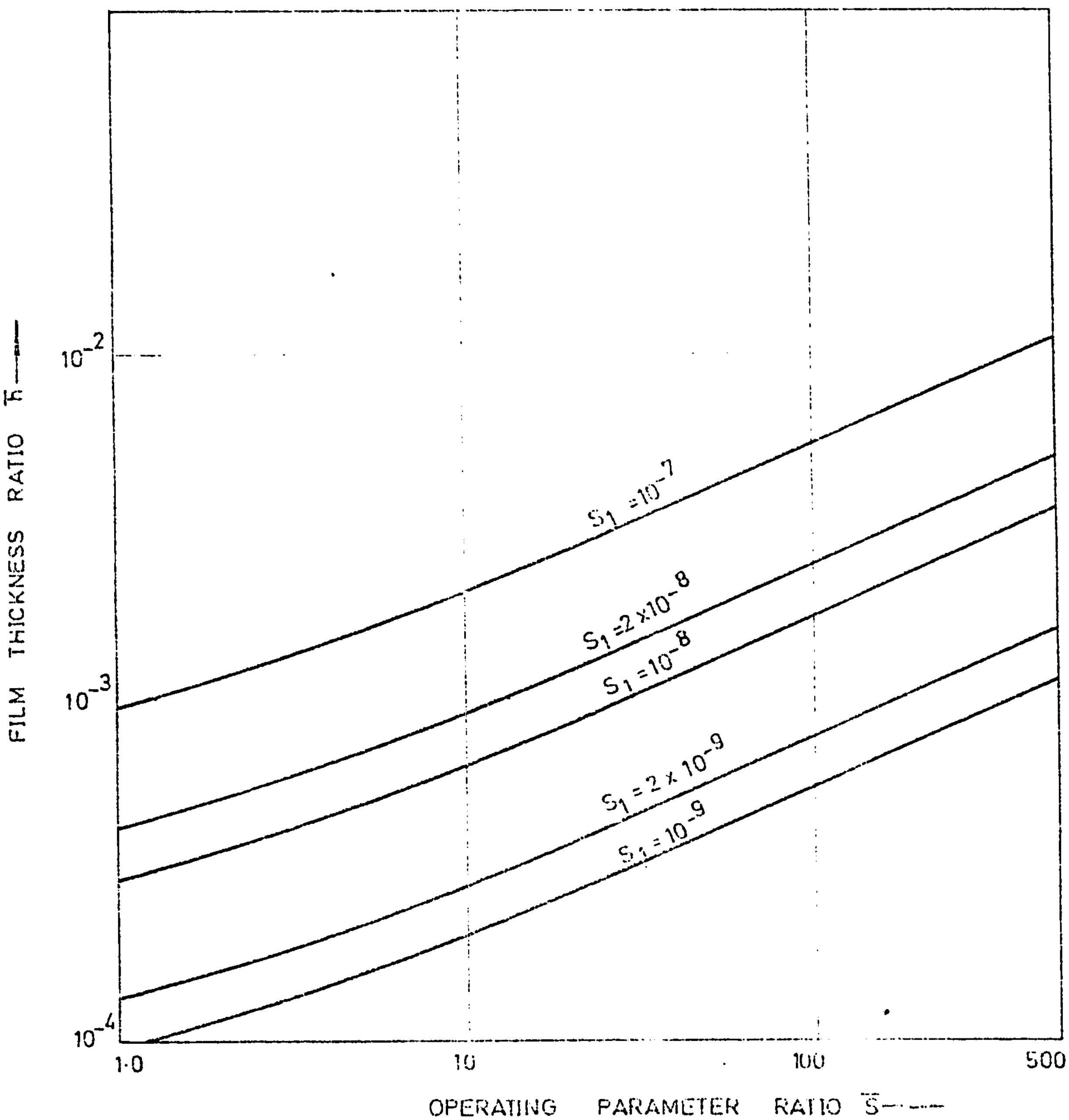

FIGURE 11 – VARIATION OF FILM THICKNESS RATIO $\bar{h}$ WITH OPERATING PARAMETER RATIO $\bar{S}$. (N = 16)

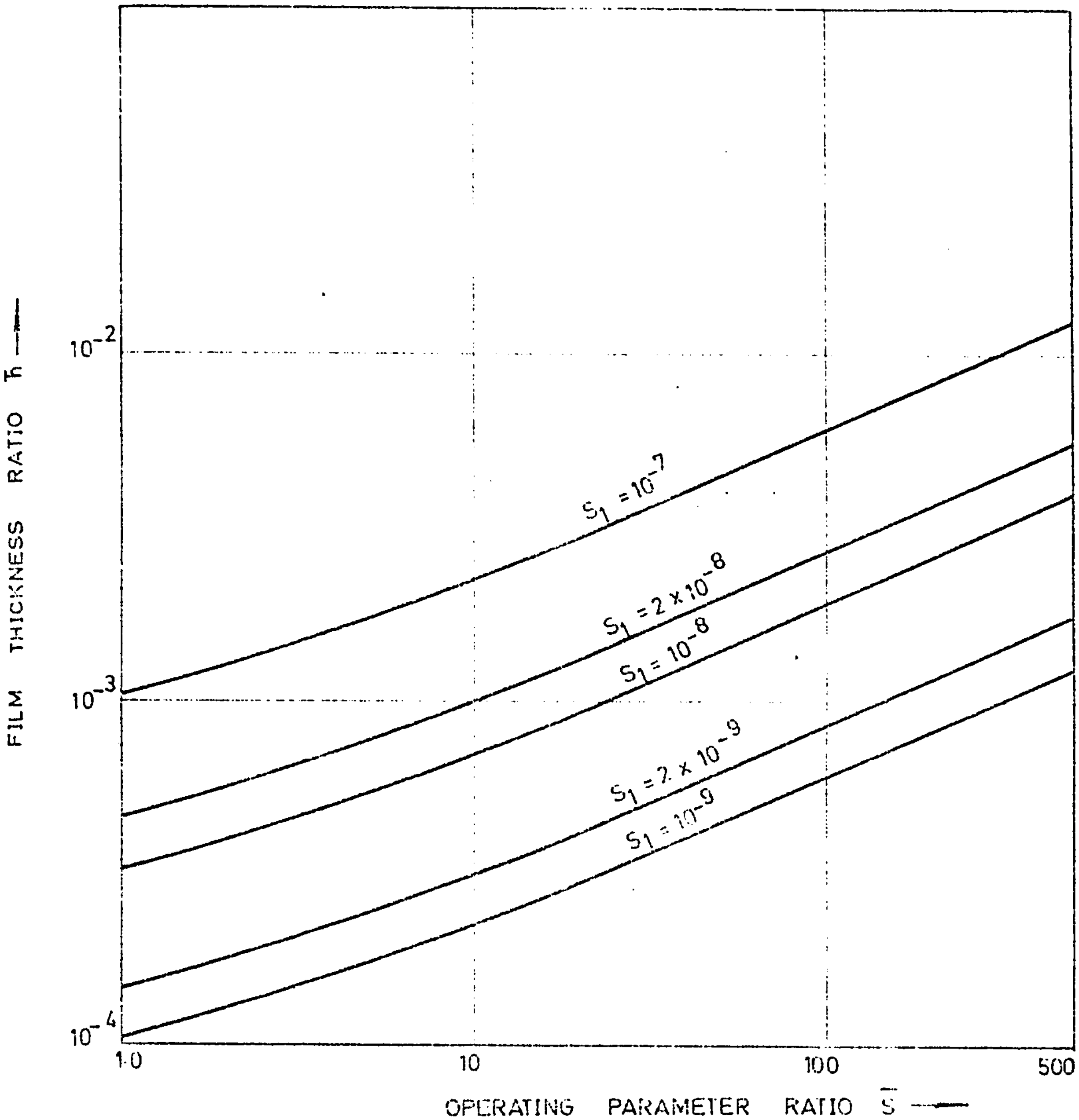

FIGURE 12 _ VARIATION OF FILM THICKNESS RATIO $\bar{h}$ WITH OPERATING PARAMETER RATIO $\bar{S}$. (N = 20)

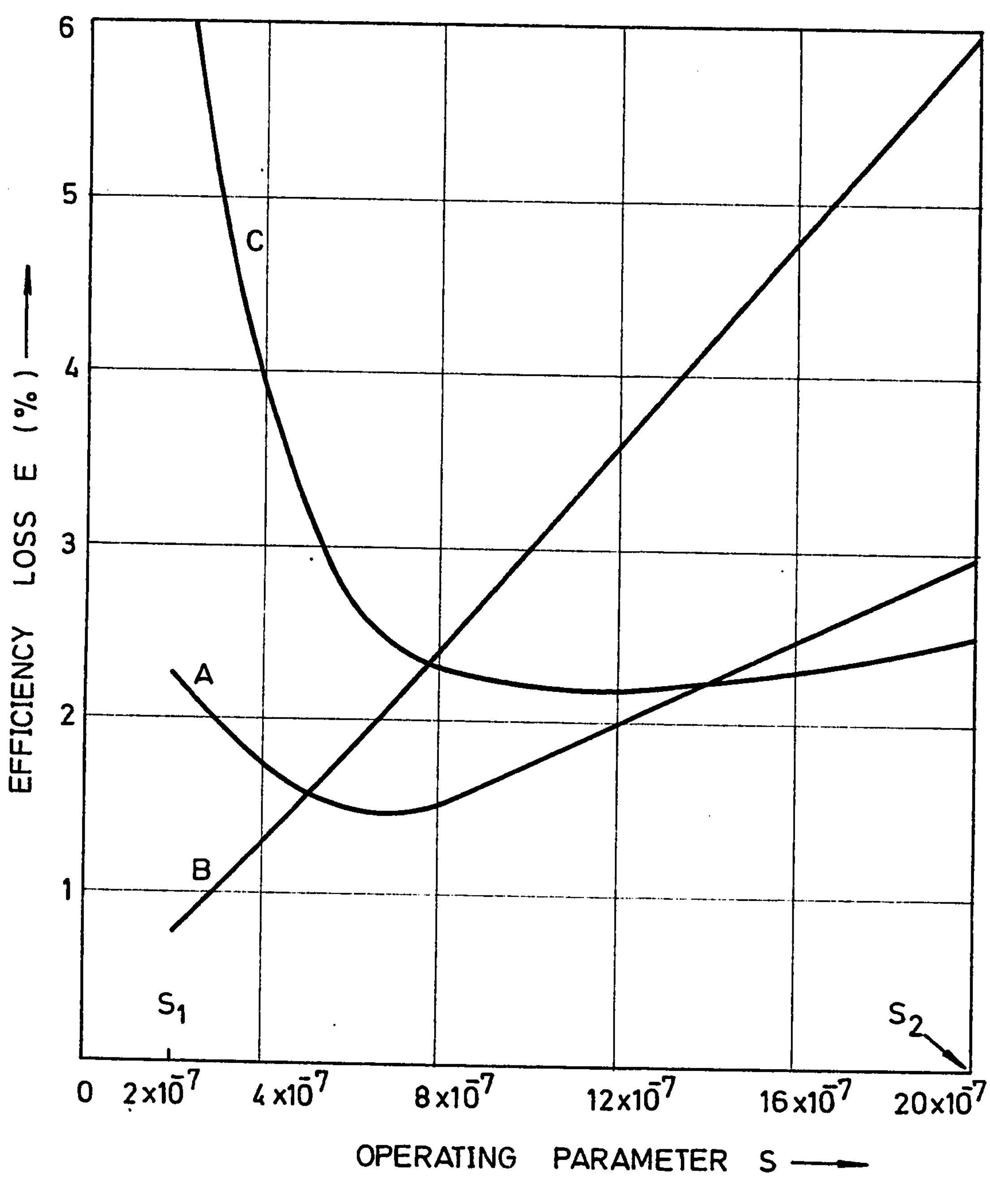

FIGURE 13- VARIATION OF POWER LOSS WITH OPERATING PARAMETER.